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Mechanism of Occurrence of Photonic Band Gap

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The density of state minimum gives a pseudogap. The minimum refractive index difference may also assure the occurrence of the photonic band gap combined with density of states minimum as it leads to overlapping of gaps for different Brillouin zone (BZ). We intend to reanalyze the situation for the occurrence of the photonic band gap taking into consideration effective dielectric constant for long wavelength limit, mechanism of minimum dielectric constant difference, optimum scattering strength and overlapping of gaps for different points of BZ. The expressions of relative width has been derived the effective dielectric constant for heterogeneous medium is experimentally observed sand witching the medium.

Lorentz-Lorentz[1], Brugmann's approach [2], Korringa, Kohn and Rostoker (KKR) band structure procedure[3], Busch and Soukoulis[4] effective medium theory have discussed the effective dielectric constant. We use Maxwell Garnet (MG)[5], multiple modified Maxwell Garnet (MMMG)[5], and effective medium (EM) theory [7] for the calculation of effective dielectric constant of a composite. The expressions used are

$$<\varepsilon>_{MG} = \varepsilon_b\,[2\varepsilon_b + \varepsilon_a + 2f\,|\varepsilon_a\text{-}\varepsilon_b| / [2\varepsilon_b + \varepsilon_a f\,|\varepsilon_a\text{-}\varepsilon_b|\,] \quad (1)$$

$$<\varepsilon>_{MMMG} = \varepsilon_b[\varepsilon_b + [P^E(1\text{-}f) + f\,]\,|(\varepsilon_a\text{-}\varepsilon_b)|\,]/[\varepsilon_b + P^E(1\text{-}f)\,|(\varepsilon_a\text{-}\varepsilon_b)|\,] \quad (2)$$

$$3(1\text{-}f)\,/\,2+\varepsilon_b/<\varepsilon>_{EM} + 3f\,/\,2+\varepsilon_a\,/<\varepsilon>_{EM} = 1 \quad (3)$$

where f is the volume filling fraction, ε_a is dielectric constant of dielectric material while ε_b is that for the surrounding medium. The composite material may be of two types i.e. dielectric sphere ($\varepsilon_a=\varepsilon,\ \varepsilon_b=1$) and dielectric air atom ($\varepsilon_a=1,\ \varepsilon_b=\varepsilon$). The P^E is the depolarizing factor, which has been taken $\approx 1/3$ in the present calculation. The present variation of effective dielectric constant versus filling fraction is given in Fig. 1 for dielectric sphere and for spherical air atom using different approaches.

We consider the effective dielectric constant of the following form

$$<\varepsilon> = f\,\varepsilon_a + (1\text{-}f)\,\varepsilon_b. \quad (4)$$

Applying the condition of the minimum value of effective dielectric constant i.e. $f\,\varepsilon_a = (1\text{-}f)\,\varepsilon_b$, we get optimum filling fraction $f_{opt}=\varepsilon_b/(\varepsilon_a+\varepsilon_b)$. We assumed the scattering strength as $\varepsilon_r=(\varepsilon_a-\varepsilon_b)/(\varepsilon_a+\varepsilon_b)$ and the optimum value of the scattering strength could be given as $\varepsilon_{ropt}=(\varepsilon_a-\varepsilon_b)/\,2\sqrt{(\varepsilon_a\varepsilon_b)}$.

Here we have assumed the arithmetical mean $(\varepsilon_a + \varepsilon_b)/2$ equal to the geometrical mean $\sqrt{(\varepsilon_a \varepsilon_b)}$. Using this result the relative width at W-point of the BZ is

$$\Delta\omega/\omega_w = 2[\{(5-\varepsilon_{ropt})^{-1/2} - (5+\varepsilon_{ropt})^{-1/2}\}/\{(5-\varepsilon_{ropt})^{-1/2} + (5+\varepsilon_{ropt})^{-1/2}\}] \quad (5)$$

Where ratio of propagation vector (**k**) and reciprocal vector (**G**) is $(5/12).^{1/2}$ The value of relative width for $\varepsilon_{ropt} = 1$ is 20.2%. The overlapping of gaps is also crucial to assure existence of the PBG. Let us consider X- and L- point of the BZ which differ in frequency such that $\nu_x/\nu_L = (2/\sqrt{3})$ giving a gap $(\nu_x - \nu_L)$ which could be normalized to one of these frequencies therefore $\Delta\nu/\nu_x$ where $\Delta\nu = (\nu_x - \nu_L)$. For overlapping of gaps we get $(\Delta\omega/\omega_w)$ equal to $\Delta\omega/\omega_x$, i.e.,

$$(1-\sqrt{3}/2) = 2[\{(5\varepsilon_0-\varepsilon_1)^{-1/2} - (5\varepsilon_0+\varepsilon_1)^{-1/2}\}/\{(5\varepsilon_0-\varepsilon_1)^{-1/2} + (5\varepsilon_0+\varepsilon_1)^{-1/2}\}] \quad (6)$$

Which gives $\varepsilon_1/\varepsilon_0 \approx 0.335$ and it is just more than 1/3 as assumed by John [8] for the occurrence of this pseudo PBG. The present work provides useful ideas about variation of effective dielectric constant as a function of filling fraction. The occurrence of the PBG could be assured by the mechanisms of minimum density of states optimum scattering strength minimum refractive index difference and the overlapping of gaps for different BZ.

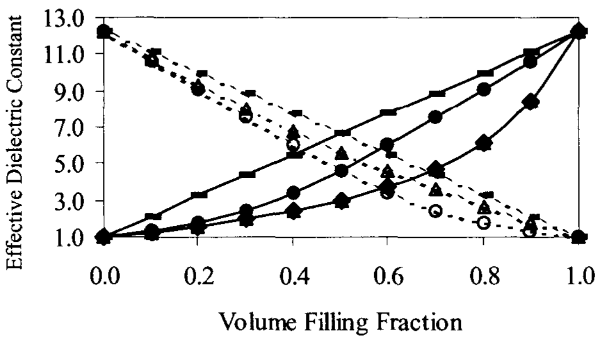

Fig. 1. Variation of effective dielectric constant with filling fraction for spherical air atoms (◆) MG result, (▲) MMMG result (●) EM theory, (−) Experimental result [7], and for dielectric sphere (◊) MG result, (△) MMMG result, (o) EM theory, (-) Experimental result.

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